Discussions on Stability of Diquarks

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Abstract

Since the birth of the quark model, the diquark which is composed of two quarks has been considered as a substantial structure of color anti-triplet. This is not only a mathematical simplification for dealing with baryons, but also provides a physical picture where the diquark would behave as a whole object. It is natural to ask whether such a structure is sufficiently stable against external disturbance. The mass spectra of the ground states of the scalar and axial-vector diquarks which are composed of two-light (L-L), one-light-one-heavy (H-L) and two-heavy quarks (H-H) respectively have been calculated in terms of the QCD sum rules. We suggest a criterion as the quantitative standard for the stability of the diquark. It is the gap between the masses of the diquark and $\sqrt{s_0}$ where s_0 is the threshold of the excited states and continuity, namely the larger the gap is, the more stable the diquark would be. In this work, we calculate the masses of the type H-H to complete the series of the spectra of the ground state diquarks. However, as the criterion being taken, we find that all the gaps for the various diquaks are within a small range, especially the gap for the diquark with two heavy quarks which is believed to be a stable structure, is slightly smaller than that for other two types of diquarks, therefore we conclude that because of the large theoretical uncertainty, we cannot use the numerical results obtained with the QCD sum rules to assess the stability of diquarks, but need to invoke other theoretical framework.

PACS numbers: 11.55.Hx, 12.38.Lg

1 Introduction

Right after the birth of the quark model, the diquark model was proposed: two quarks constitute a color-anti-triplet which may be a tightly bound state. In Gell-Mann's pioneer paper on the quark model, he discussed the possibility of existence of free diquarks[1].

With the diquark picture, numerous authors have studied the processes where baryons are involved [2, 3, 4, 5, 6] and their conclusions support the existence of diquarks. Even in the meson sector, some newly observed resonances are considered to possess the exotic structures. One possibility is that the mesons are tetraquarks composed of a diquark and an anti-diquark, [7, 8, 9, 10, 11]. In fact, it is still in dispute that the diquark is a real spatially bound state as a pseudo-particle or just a loosely bound state. Recently, the authors of Refs.[12, 13] treat the diquark as an explicit particle which is the essential ingredient inside hadrons (baryons or exotic mesons). If these diquarks indeed exist as stable particles, they should have certain and definite masses and quantum numbers. Just as we discuss the regular hadrons, their spectra not only possess the real part, but also the imaginary part which corresponds to the stability of the diquark. Namely, the lifetime of the diquark might be finite. The main scenario to determine the diquark lifetime is that with an external disturbance, the diquark might dissolve into two quarks. By our intuition, the diquark composed of two light quarks might be easier to dissolve by absorbing gluons. Generally, it is believed that the heavy diquarks which are composed of two heavy quarks are more stable against external disturbance.

In principle, a baryon which is composed of three valence quarks is described by the Faddeev equation group composed of three coupled differential equations[14, 15]. For the equations, the three valence quarks are of the same weights. It is possible that two of the three quarks would accidentally constitute a bound state, say, by quantum fluctuation. The diquark can be treated as a sub-system and behaves as an independent particle. In this picture the baryon possesses a diquark-quark structure and thus the three-body system turns into a two-body one. Correspondingly, the three Faddeev equations reduce to a single equation (no mater relativistic or non-relativistic). Therefore the problem is greatly simplified. It is noted on other side, that the sub-system is not exactly a fundamental one, but possesses an inner structure. When it interacts with gluons, a form factor which manifests the inner structure should be introduced. Now, we are confronting a problem: which two valence quarks of the three would tend to combine into a bound state. In fact, any two quarks have a chance to combine via strong interactions, but the rest valence quark would interact with the individual quarks in the diquark and tend to

tear it apart. Thus the key point is if such sub-system is sufficiently stable against the disturbance. When we deal with the baryons which are composed of three light quarks, one-heavy-two-light quarks and two-heavy-one-light quarks, we notice an obvious unequal structures. We need to determine which type of diquark i.e. the diquarks with light-light (L-L), heavy-light (H-L) or heavy-heavy (H-H) structures, is more stable. Thus we can more confidently reduce the three Faddeev equations to one with the expected diquark subsystem. For serving this aim, it is significant to investigate the stability of diquarks.

First of all, it is important and interesting to investigate the spectra of the diquarks with various quark contents. According to the masses of different flavors, one can categorize the diquarks into three types: light-light diquark (L-L), light-heavy diquark (H-L) and heavy-heavy diquark (H-H), where light quarks are u, d, s and heavy quarks are c, b.

The QCD is by all means a successful theory for the strong interaction, but the non-perturbative QCD which dominates the low energy phenomena is still not fully understood yet. Among the theoretical methods for treating the non-perturbative QCD effects, the QCD Sum Rules[16] is believed to be a powerful means for evaluating the hadronic spectra and other properties of hadrons. In Refs.[10, 17], the mass spectra of the scalar light-light diquark states (L-L) were studied with the QCD sum rules. Recently, with the same method, Wang studied the light-heavy diquark states (H-L)[18].

By the common sense, the diquark composed of two heavy quarks (H-H) may be a kinematically favorable sub-structure in baryon. For studying the stability of the three different types (L-L, H-L and H-H) diquarks, it would be crucial to discuss their properties in a unique framework. In this work, we are going to carry out the job in terms of the QCD sum rules. Then we will discuss the feasibility of such a scheme by scanning the numerical results obtained in this method.

In this paper, we calculate the masses of heavy-heavy diquark states (H-H) with the QCD sum rules. Then combining the results presented in the relevant works [17, 18], we discuss the stability of the diquarks altogether.

Obviously it is crucial to set a reasonable and practical criterion of the stability of the sub-structures − diquarks.

In the scenario of the QCD sum rules, numerically there exists a threshold s_0 corresponding to a starting point beyond which higher excited and continuous states reside. This cutoff provides a natural criterion which we may use to study the stability of the diquark. Namely, according to our general knowledge on quantum mechanics, the continuous spectra would correspond to the dissolved state where the constituents of the supposed-bound state would be set free. Thus we choose the gap between the ground state and the corresponding threshold $\sqrt{s_0}$ as the criterion of stability of the diquarks.

However, as well known that there is a 20% theoretical uncertainty in all the computations via the QCD sum rules, therefore, even though such criterion may be indeed reasonable in principle, it is still doubtful if the results obtained in terms of the QCD sum rules can practically apply to reflect the diquark stability. The goal of this work is to testify the reasonability of applying the supposed criterion within the framework of the QCD sum rules.

In the last section, we will come back to discuss the feasiability based on the numerical results we obtained in term of the QCD sum rules.

The paper is organized as follows. After the introduction, in Sec.II we derive the correlation function of the suitable currents with proper quantum numbers in terms of the QCD sum rules. In Sec. III, our numerical results and relevant figures are presented. Section IV is devoted to a summary and concluding remarks. The tedious analytical results are collected in the appendices.

2 Formalism

For studying the scalar and axial-vector H-H diquark with the QCD sum rules, we write down the local color anti-triplet diquark currents:

$$J^i(x) = \epsilon^{ijk} Q_j^T(x) C\gamma^5 Q_k(x) \ , \qquad (1)$$

$$J^i_\mu(x) = \epsilon^{ijk} Q_j^T(x) C\gamma_\mu Q_k(x) \ , \qquad (2)$$

where i, j, k are the color indexes, $Q = b, c$, and C is the charge conjugation operator.

In order to perform the QCD sum rules, we define the two-point correlation functions $\Pi(q)$ (for scalar diquark) and $\Pi_{\mu\nu}(q)$ (for axial-vector diquark) as follows:

$$
\begin{aligned}
\Pi(q) &= i\int d^4x e^{iq\cdot x}\langle 0|T\Big\{J^i(x)J^{i\dagger}(0)\Big\}|0\rangle \ , &(3)\\
\Pi_{\mu\nu}(q) &= i\int d^4x e^{iq\cdot x}\langle 0|T\Big\{J^i_\mu(x)J^{i\dagger}_\nu(0)\Big\}|0\rangle \ . &(4)
\end{aligned}
$$

On the hadron side, after separating out the ground state contribution from the pole terms, the correlation function is expressed as a dispersion integral over the physical regime,

$$
\begin{aligned}
\Pi(q) &= \frac{\lambda_S^2}{M_S^2-q^2}+\frac{1}{\pi}\int_{s_S^0}^{\infty} ds\frac{\rho_S^h(s)}{s-q^2} \ , &(5)\\
\Pi_{\mu\nu}(q) &= \Big(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\Big)\Big\{\frac{\lambda_A^2}{M_A^2-q^2}+\frac{1}{\pi}\int_{s_A^0}^{\infty} ds\frac{\rho_A^h(s)}{s-q^2}\Big\} \ , &(6)
\end{aligned}
$$

where M_t with subscript t being S or A for the scalar or axial-vector respectively, is the mass of the ground state diquark, $\rho_t^h(s)$ is the spectral density and represents the contribution from the higher excited states and the continuum, s_t^0 is the threshold for the excited states and continuum, the pole residues λ_t correspond to the diquark coupling strength, is defined through[18]:

$$
\begin{aligned}
\langle 0|J^i(0)|S^j(q)\rangle &= \lambda_S\delta^{ij} \ , &(7)\\
\langle 0|J^i_\mu(0)|A^j(q)\rangle &= \lambda_A\epsilon_\mu\delta^{ij} \ , &(8)
\end{aligned}
$$

with ϵ_μ being the polarization vector of the axial-vector diquark.

On the quark side, the operator product expansion (OPE) is applied to derive the correlation functions. Firstly, we can write down the "full" propagator $S_Q^{ij}(x)$ of a massive quark, where the vacuum condensates are clearly displayed[19].

$$
\begin{aligned}
S_Q^{ij}(x) &= \frac{i}{(2\pi)^4}\int d^4p e^{-ip\cdot x}\Bigg\{\frac{\delta_{ij}}{\not p-m_Q}-\frac{g_s(t^k)^{ij}G_k^{\alpha\beta}}{4}\frac{\sigma_{\alpha\beta}(\not p+m_Q)+(\not p+m_Q)\sigma_{\alpha\beta}}{(p^2-m_Q^2)^2}\\
&\quad +\frac{\pi^2}{3}\langle\frac{\alpha_s GG}{\pi}\rangle\delta_{ij}m_Q\frac{p^2+m_Q\not p}{(p^2-m_Q^2)^4}+\cdots\Bigg\} \ , &(9)
\end{aligned}
$$

where $\langle\frac{\alpha_s GG}{\pi}\rangle=\langle\frac{\alpha_s G_{\alpha\beta}G^{\alpha\beta}}{\pi}\rangle$, then contracting the quark fields in the correlation functions, we gain the results:

$$\Pi(q) = -i\epsilon^{ijk}\epsilon^{ij'k'}\int d^4x e^{iq\cdot x} Tr\Big\{\gamma_5 S_Q^{jj'}(x)\gamma_5 C S_Q^{kk'T}(x) C\Big\}\ , \tag{10}$$

$$\Pi_{\mu\nu}(q) = i\epsilon^{ijk}\epsilon^{ij'k'}\int d^4x e^{iq\cdot x} Tr\Big\{\gamma_\mu S_Q^{jj'}(x)\gamma_\nu C S_Q^{kk'T}(x) C\Big\}\ . \tag{11}$$

Then substituting the full c and b quark propagators into above correlation functions and integrating over the variable k, we obtain the correlation functions at the level of quark-gluon degrees of freedom. Simply, the correlation function $\Pi_t(q^2)$ (t=S or A) is written as:

$$\Pi_t(q) = \Pi_t^{\rm pert}(q) + \Pi_t^{\rm cond,4}(q)\ , \tag{12}$$

where the superscripts "pert", and"cond" refer to the contribution from the perturbative QCD, and gluon condensates, respectively. In this work, we only keep the two-gluon condensate in consideration for the heavy quark condensates are zero as suggested in literature [19].

Due to the quark-hadron duality, we differentiate Eq.(14) with respect to $\frac{1}{M_B^2}$, then eliminate the pole residues λ_t, and obtain the resultant sum rule for the mass spectra of the H-H diquark states:

$$M_t = \sqrt{-\frac{R_t^1}{R_t^0}}\ , \tag{13}$$

with

$$R_t^0 = \frac{1}{\pi}\int_{(m_{Q_1}+m_{Q_2})^2}^{s_t^0} ds\rho_t^{\rm pert}(s)e^{-s/M_B^2} + \hat{\mathbf{B}}[\Pi_t^{\rm cond,4}(q^2)]\ , \tag{14}$$

$$R_t^1 = \frac{\partial}{\partial M_B^{-2}} R_t^0\ . \tag{15}$$

Here, M_B is the Borel parameter and s_0 is the threshold cutoff introduced to remove the contribution of the higher excited and continuum states [20, 21].

For the H-H diquark states, the detailed expressions of R_t^0 are collected in the appendix.

3 Numerical Analysis

3.1 The masses of the ground diquark states with only heavy flavors.

The numerical parameters used as inputs in this work are taken as [16, 18, 19, 22]

$$\begin{aligned}
&\langle\bar{q}q\rangle = -(0.24 \pm 0.01\text{GeV})^3 \ , && \langle\bar{s}s\rangle = (0.08 \pm 0.02)\langle\bar{q}q\rangle \ , \\
&\langle\bar{q}g_s\sigma Gq\rangle = m_0^2\langle\bar{q}q\rangle \ , && \langle\bar{s}g_s\sigma Gs\rangle = m_0^2\langle\bar{s}s\rangle \ , \\
&m_0^2 = (0.8 \pm 0.2)\text{GeV}^2 \ , && \langle\frac{\alpha_s}{\pi}G^2\rangle = 0.012\text{GeV}^4 \ , \\
&m_u \simeq m_d = 0.005\text{GeV} \ , && m_s = (0.14 \pm 0.01)\text{GeV} \ , \\
&m_c = (1.35 \pm 0.10)\text{GeV} \ , && m_b = (4.7 \pm 0.1)\text{GeV} \ .
\end{aligned} \tag{16}$$

where the energy scale is $\mu = 1$ GeV.

It is crucial to determine the proper threshold s^0 and Borel parameter M_B^2 for obtaining physical spectra. For justifying if the choice is suitable, there are two criteria. First, the perturbative contribution should be larger than the contributions from all kinds of condensates, and another is that the pole contribution should be larger than the continuum contribution[16, 19]. In our work, the error bars are estimated by varying the Borel parameters, s^0 and including the uncertainties of the input parameters as well.

	mass(GeV)	M_B^2(GeV^2)	$\sqrt{s^0}$(GeV)	pole	$\langle\frac{\alpha_s GG}{\pi}\rangle$
$cc(1^+)$	2.99 ± 0.10	$1.2 - 2.5$	3.3 ± 0.1	$(56 - 88)\%$	$(4 - 8)\%$
$bc(0^+)$	6.30 ± 0.09	$2.2 - 5.0$	6.6 ± 0.1	$(85 - 99)\%$	$(5 - 14)\%$
$bc(1^+)$	6.36 ± 0.08	$3.0 - 6.0$	6.7 ± 0.1	$(53 - 85)\%$	$(3 - 5)\%$
$bb(1^+)$	9.76 ± 0.08	$6.0 - 14.0$	10.1 ± 0.1	$(41 - 79)\%$	$(0.04 - 0.23)\%$

Table 1: For the H-H diquark states, we show the masses, the preferred Borel parameters M_B^2, the threshold parameters s^0, the contribution from the pole term to the spectral density and the contribution from $\langle\frac{\alpha_s}{\pi}G^2\rangle$.

The spectra of the L-L and H-L diquarks have been studied in the previous literature[17, 18]. Our input parameters are the same as that used in Ref.[18]. Numerically we have rechecked the results of the previous works for spectra of L-L and H-L diquarks and find

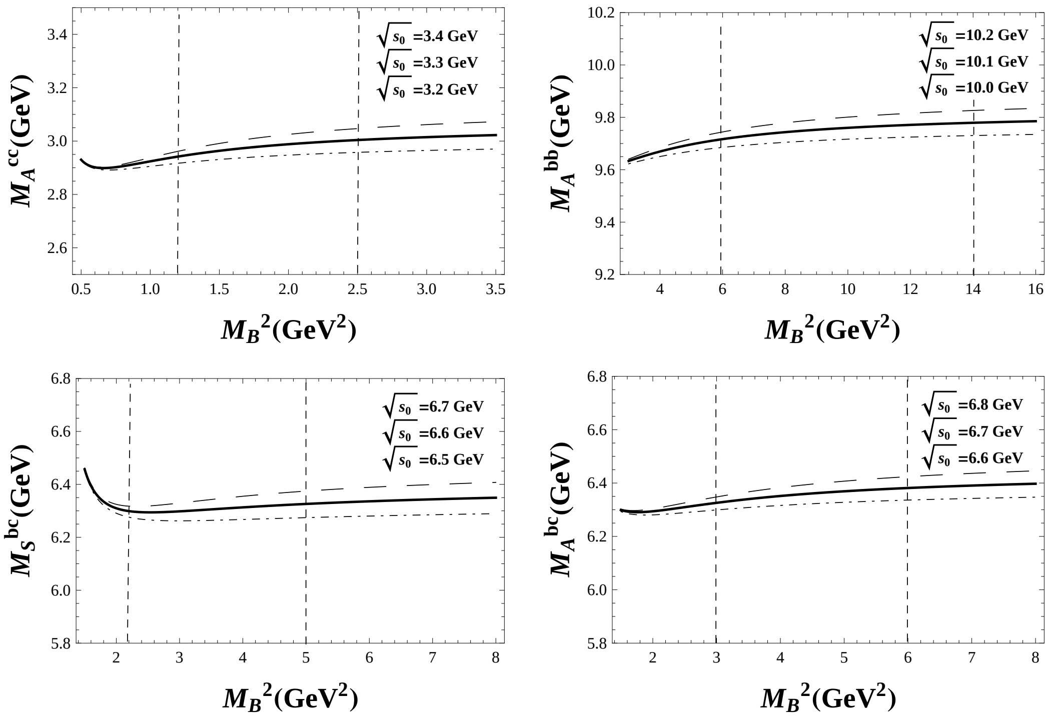

Figure 1: Dependence of M_A^{cc}, M_A^{bb}, M_S^{bc} and M_A^{bc} on the Borel parameter M_B^2. We deliberately put two vertical lines denoting the chosen Borel window.

our results are consistent with them, thus we simply present the relevant results for the L-L and H-L diquarks in the following table. The analytical formulations for the H-H diquarks are shown in the appendix and that for L-L and H-L can be found in the relevant references which we list in the bibliography of the work.

3.2 Discussion on the stability of the ground diquark states.

In Table1, we present the masses and the threshold $\sqrt{s_0}$ of the H-H scalar and axial-vector diquark ground states. Then, altogether with that given in literature, we collect the results for all these three types of diquark in the following table. Here we define the energy gap as $\Delta E = \sqrt{s_0} - M_d$, where M_d is the mass of the corresponding diquark state.

As aforementioned, we use energy gap ΔE to embody the stability of diquarks. Namely, according to our general knowledge on quantum mechanics, the continuous spectra would correspond to a dissolved state where the constituents of the supposed-bound state are set

	mass(GeV)	M_B^2(GeV^2)	$\sqrt{s^0}$(GeV)	ΔE(GeV)
$sq(0^+)$	0.55 ± 0.03	1.0	0.95	0.40
$qq(1^+)$	0.34 ± 0.04	4.0	0.85	0.51
$sq(1^+)$	0.42 ± 0.03	4.0	0.95	0.53
$ss(1^+)$	0.50 ± 0.05	4.0	1.05	0.55

Table 2: The diquark masses, the preferred Borel parameters M_B, the preferred threshold parameters s^0 obtained by choosing reasonable plateaus, and the energy gap ΔE for each possible L-L diquark state. Here the revelent results are derived from the corresponding formulae in Ref.[17] with our input parameters.

	mass(GeV)	M_B^2(GeV^2)	$\sqrt{s^0}$(GeV)	ΔE(GeV)
$cq(0^+)$	1.77	1.50	2.19	0.42
$cq(1^+)$	1.76	1.60	2.19	0.43
$cs(0^+)$	1.84	1.55	2.24	0.40
$cs(1^+)$	1.84	1.65	2.24	0.40
$bq(0^+)$	5.14	3.85	5.48	0.34
$bq(1^+)$	5.13	3.95	5.48	0.35
$bs(0^+)$	5.20	3.95	5.57	0.37
$bs(1^+)$	5.22	4.10	5.57	0.35

Table 3: The diquark masses, the preferred Borel parameters M_B, the preferred threshold parameters s^0 obtained by choosing reasonable plateaus for the H-L diquarks. These revelent results have already been obtained in Ref.[18].

free. Thus we choose the gap between the mass of the ground state and the corresponding threshold $\sqrt{s_0}$ as the criterion of stability of the diquark. In Tables(2-4), we show the energy gap ΔE for the three types of diquarks. Concretely, the energy gap is between 0.40 GeV and 0.55 GeV for L-L diquarks, between 0.34 GeV and 0.43 GeV for H-L diquarks, and between 0.30 GeV and 0.34 GeV for H-H diqaurks. One can observe that the sequence of the energy gaps for these three types of diquarks is :$\Delta E_{L-L} > \Delta E_{H-L} > \Delta E_{H-H}$. If larger energy gap implies more stable structure, the L-L diquarks should be the stablest one and the H-H diquarks is the most instable structure, however, this definitely contradicts to our intuition. Let us discuss this issue in next section.

	mass(GeV)	$M_B^2(\text{GeV}^2)$	$\sqrt{s^0}(\text{GeV})$	$\Delta E(\text{GeV})$
$bc(0^+)$	6.30	3.6	6.6	0.30
$cc(1^+)$	2.99	1.9	3.3	0.31
$bc(1^+)$	6.36	4.5	6.7	0.34
$bb(1^+)$	9.76	10.0	10.1	0.34

Table 4: The H-H diquark masses, the preferred Borel parameters M_B, the preferred threshold parameters s^0 obtained by choosing reasonable plateaus.

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4 Summary and Conclusions

In this work, we try to study the stability of scalar and axial vector diquarks of three types L-L, H-L and H-H in terms of the QCD sum rules. For the purpose, we first calculate the mass spectra of the scalar (0^+) and axial-vector (1^+) H-H diquarks. Together with the results given in the previous works about the L-L diquarks [17] and the H-L diquarks[18], we can compare the corresponding ΔE which is supposed to be a reference criterion for the stability of the diquark sub-structure.

However, the numerical results obtained in terms of the QCD sum rules determine the sequence $\Delta E_{L-L} > \Delta E_{H-L} > \Delta E_{H-H}$. Even though actually the values of ΔE_{L-L}, ΔE_{H-L} and ΔE_{L-L} are not far apart, this sequence obviously conflicts to our common sense where the H-H diquark is the most stable structure.

Because the computation with the QCD sum rules possess over 20% errors and the determined ΔE values do not differ much (less than 20%), we cannot decide that the H-H diquark is the most instable one.

Thus, we would be tempted to conclude that due to the large errors brought up in the computations with the QCD sum rules, it is not suitable to determine the stability in terms of the QCD sum rules, and we have to turn to other theoretical approaches to investigate this important issue which would be the goal of our next work.

Acknowledgments

When we complete the work, we notice that a new paper about the spectra of the L-L diquark appears at the ArXiv as hep-ph 1112.5910. Even though the concrete numbers are slightly different from that given in Ref.[15], the general trend is similar and our qualitative conclusion does not change at all. This work was supported in part by the National Natural Science Foundation of China(NSFC) under contract N0.11075079.

Appendix

For the H-H diquark, our analytical expressions are shown as follows:

$$\rho_S(s) = -\frac{3}{4\pi s}\left(m_{Q_1}^2 + 2m_{Q_1}m_{Q_2} + m_{Q_2}^2 - s\right)\sqrt{(m_{Q_1}^2 - m_{Q_2}^2 + s)^2 - 4m_{Q_1}^2 s}\,, \quad (17)$$

$$\rho_A(s) = \frac{3}{2\pi s}\left(m_{Q_1}^2 - 4m_{Q_1}m_{Q_2} + m_{Q_2}^2\right)\sqrt{(m_{Q_1}^2 - m_{Q_2}^2 + s)^2 - 4m_{Q_1}^2 s}\,, \quad (18)$$

$$\begin{aligned}
G_S^1(M_B^2) &= \langle\alpha_s G^2\rangle \int_0^1 dx e^{-\frac{\frac{m_{Q_1}^2}{x}+\frac{m_{Q_2}^2}{1-x}}{M_B^2}} \Bigg\{ \frac{(4-9x)(1-x)}{16\pi} + \frac{1}{32\pi x^3 M_B^2}\Big[2(1-x)(9x^2 \\
&\quad -13x+4)x^3 m_{Q_1}^2 + 18x m_{Q_1}m_{Q_2} + 2(x-1)(9x-4)x^4 m_{Q_2}^2\Big] + \frac{1}{32\pi x^3 M_B^4} \\
&\quad \times\Big[(1-x)(-9x^5+22x^4-17x^3+4x^2-7x+7)x m_{Q_1}^4 + (6x^3-6x^2+3x \\
&\quad -2)m_{Q_1}^3 m_{Q_2} + (1-x)\left(18x^4-26x^3+8x^2+7\right)x^2 m_{Q_1}^2 m_{Q_2}^2 - 6x^3 m_{Q_1} m_{Q_2}^3 \\
&\quad +(4-9x)(1-x)x^5 m_{Q_2}^4\Big] + \frac{1}{32\pi x^3 M_B^6}\Big[3(1-x)(x-1)^2\left(x^3-x^2+1\right)x^2 \\
&\quad \times m_{Q_1}^6 + 3\left(x^4-2x^3+x^2+x-1\right)x m_{Q_1}^5 m_{Q_2} + 3(x-1)(3x^4-6x^3+3x^2 \\
&\quad +2x-2)x^3 m_{Q_1}^4 m_{Q_2}^2 - 3\left(2x^3-2x^2+1\right)x^2 m_{Q_1}^3 m_{Q_2}^3 + (1-x)(9x^3-9x^2 \\
&\quad +3)x^4 m_{Q_1}^2 m_{Q_2}^4 + 3x^5 m_{Q_1} m_{Q_2}^5 + 3(x-1)x^7 m_{Q_2}^6\Big]\Bigg\}\,, \qquad (19a) \\
G_S^2(M_B^2) &= G_1^S(M_B^2, m_{Q_1} \leftrightarrow m_{Q_2})\,, \qquad (19b)
\end{aligned}$$

$$
\begin{aligned}
G^3_S(M_B^2) &= \langle\alpha_s G^2\rangle \int_0^1 dx e^{-\frac{\frac{m_{Q_1}^2}{x}+\frac{m_{Q_2}^2}{1-x}}{M_B^2}} \Big\{ -\frac{9\left(2x^2-2x-1\right)}{64\pi} - \frac{1}{64\pi(x-1)^3x^3M_B^2} \\
&\times\Big[3(3x^2\left(2x^2-2x-1\right)(x-1)^3m_{Q_1}^2 + 8x^2(x-1)^2m_{Q_1}m_{Q_2} \\
&-3x^3\left(2x^2-2x-1\right)(x-1)^2m_{Q_2}^2)\Big] - \frac{1}{64\pi(x-1)^3x^3M_B^4} \\
&\times\Big[3(3(x-1)^2x^4m_{Q_2}^4 + 3(x-1)^4x^2m_{Q_1}^4 + (x-1)x^2(-6x^3+12x^2 \\
&-7x+1)m_{Q_2}^2m_{Q_1}^2)\Big] - \frac{1}{64\pi(x-1)^3x^3M_B^6}\Big[3(-x^5m_{Q_2}^6 + (x-1)x^3 \\
&(3x-2)m_{Q_2}^4m_{Q_1}^2 + (x-1)^5m_{Q_1}^6 + (1-3x)(x-1)^3xm_{Q_2}^2m_{Q_1}^4)\Big]\Big\} \,. \qquad (20a)
\end{aligned}
$$

$$
\begin{aligned}
G^1_A(M_B^2) &= \langle\alpha_s G^2\rangle \int_0^1 dx e^{-\frac{\frac{m_{Q_1}^2}{x}+\frac{m_{Q_2}^2}{1-x}}{M_B^2}} \Bigg\{ -\frac{(x-1)(9x-4)}{8\pi} - \frac{1}{16\pi(x-1)^2x^3M_B^2}\Big[-2x^3 \\
&\times(9x-4)(x-1)^2m_{Q_2}^2 + 2x^2(9x-4)(x-1)^3m_{Q_1}^2 + 36x(x-1)^2m_{Q_1}m_{Q_2}\Big] \\
&-\frac{1}{16\pi(x-1)^2x^3M_B^4}\Big[(x-1)x^3(9x-4)m_{Q_2}^4 + (x-1)^3\left(9x^2-4x-7\right)m_{Q_1}^4 \\
&-(x-1)^2x\left(18x^2-8x-7\right)m_{Q_2}^2m_{Q_1}^2 + 12(x-1)x^2m_{Q_2}^3m_{Q_1} - 2(x-1)^2 \\
&\times(3x+2)m_{Q_2}m_{Q_1}^3\Big] - \frac{1}{16\pi(x-1)^2x^3M_B^6}\Big[-3x^4m_{Q_2}^6 + 6x^3m_{Q_2}^5m_{Q_1} \\
&+(x-1)x^2(9x-3)m_{Q_2}^4m_{Q_1}^2 + 3(x-1)^4m_{Q_1}^6 + 6(x-1)^3m_{Q_2}m_{Q_1}^5 \\
&-3(x-1)^2x(3x-2)m_{Q_2}^2m_{Q_1}^4 - 6(x-1)x(2x-1)m_{Q_2}^3m_{Q_1}^3\Big]\Bigg\} \,, \qquad (21a) \\
G^2_A(M_B^2) &= G^A_1(M_B^2, m_{Q_1} \leftrightarrow m_{Q_2}) \,, \qquad (21b)
\end{aligned}
$$

$$
\begin{aligned}
G_A^3(M_B^2) &= \langle\alpha_s G^2\rangle \int_0^1 dx e^{-\frac{\frac{m_{Q_1}^2}{x}+\frac{m_{Q_2}^2}{1-x}}{M_B^2}} \Bigg\{ -\frac{3\left(6x^2-6x+5\right)}{32\pi} - \frac{1}{32\pi(x-1)^3x^3M_B^2} \\
&\quad \times\Big[3\left((x-1)^3x^2\left(6x^2-6x+5\right)m_{Q_1}^2-(x-1)^2x^3\left(6x^2-6x+5\right)m_{Q_2}^2\right)\Big] \\
&\quad -\frac{1}{32\pi(x-1)^3x^3M_B^4}\Big[3(3(x-1)^4x^2m_{Q_1}^4+(x-1)x^2(-6x^3+12x^2-7x \\
&\quad +1)m_{Q_2}^2m_{Q_1}^2+x^3\left(3x^3-6x^2+5x-2\right)m_{Q_2}^4)\Big] - \frac{1}{32\pi(x-1)^3x^3M_B^6} \\
&\quad \times\Big[3\left((2-x)x^4m_{Q_2}^6+(x-1)x^2(3x^2-6x+2\right)m_{Q_2}^4m_{Q_1}^2+(x-1)^5m_{Q_1}^6 \\
&\quad -3(x-1)^4xm_{Q_2}^2m_{Q_1}^4)\Big]\Bigg\}\,. \qquad (22a)
\end{aligned}
$$

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